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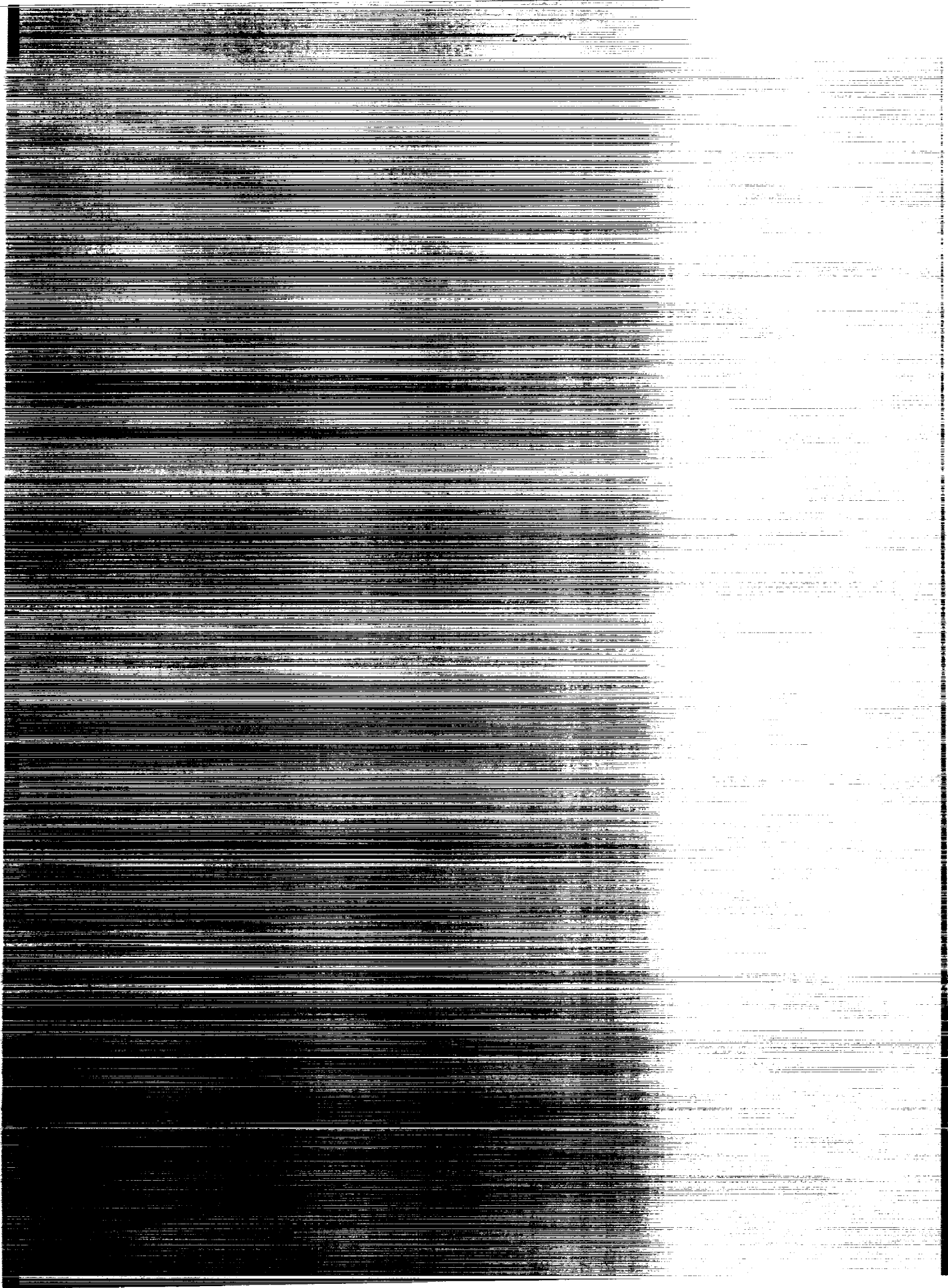
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A Finite Element Approach for the Dynamic Analysis of Joint-Dominated Structures

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ABSTRACT

A finite element method to model dynamic structural systems undergoing large rotations is presented. The dynamic systems are composed of rigid joint bodies and flexible beam elements. The configurations of these systems are subject to change due to the relative motion in the joints among interconnected elastic beams. A body fixed reference is defined for each joint body to describe the joint body's displacements. Using the finite element method and the kinematic relations between each flexible element and its corotational reference, the total displacement field of an element, which contains gross rigid as well as elastic effects, can be derived in terms of the translational and rotational displacements of the two end nodes. If one end of an element is hinged to a joint body, the joint body's displacements and the hinge degree of freedom at the end are used to represent the nodal displacements. This results in a highly coupled system of differential equations written in terms of hinge degrees of freedom as well as the rotational and translational displacements of joint bodies and element nodes.

1. Introduction

In many space applications, such as deployment of large space lattice structures, maneuvers of space crane system, slewing of space antenna, and so on, the structural systems are composed of articulated elastic links which may undergo large angular motion relative to one another. The analytical simulation of the operation of such systems is critical to their successful design since the size and gravity suspension effects limit the quality of ground test while the flight experiments are still very expensive. This analytical simulation technique is referred to as multibody dynamic analysis.

During the past decade, many efforts (Shabana and Wehage, 1984), (Singh and *et al*, 1985), (Yoo and Haug, 1986) have been devoted to the 3-D flexible multibody dynamic analysis. Because of the different characteristics of the mechanisms and/or structural systems, various assumptions and theories were adopted, and different approaches were employed to formulate the equations of motion governing the systems' behavior. Since the configuration of an elastic component in a system is defined by infinite spatial degrees of freedom; theoretically, a set of partial differential equations of time and spatial variables are required to represent the system's dynamic equations. In order to take advantage of modern computational facilities, component mode synthesis techniques, (Shabana and Wehage, 1983), (Singh and *et al*, 1985), and (Yoo and Haug, 1985), have been used to approximate the

dynamic model resulting in a set of numerically acceptable ordinary differential equations with finite spatial degrees of freedom. In most of the previous studies, the gross rigid motion of an elastic body is often defined by the translational and rotational displacements of a body reference frame which may be either fixed to or floating about the flexible body. Superimposing the linear combination of assumed modes onto the body reference frame, the total element displacement field can be defined. Using variational methods, the equations of motion of a flexible component in a system were written in terms of an uncoupled set of gross rigid and finite modal degrees of freedom. It has been shown (McGowan and Housner, 1985) that the superimposition of linear elastic model onto the non-linearly behaving reference frame can lead to erroneous predictions, when either the body is highly flexible or the rotational speed is high. Another limitation lies in the selection of appropriate mode shapes which have to be consistent with the system configuration. This selection is further complicated by the configuration changes of the dynamic systems.

In this paper, a new formulation for the transient analysis of multibody structural systems is proposed. This formulation, which overcomes the limitations stated above, is well suited to large space structures which consist of highly flexible, light weight beams. In many space structural applications, component beams may be connected together using one joint body with several hinge connections. In Fig. 1, a

typical joint body connection is shown, wherein the joint body JB comprises several hinges; through each of them, an elastic beam is connected. By adopting the finite element method, the total displacement field of each element can be defined by the three translational and three rotational displacements of the joint bodies to which the elements are hinged and the associated hinge degrees of freedom. Using the principle of virtual work (PVW), the element equations of motion can be derived in terms of joint body displacements and hinge degrees of freedom. The offset of the hinge connection to the origin of the joint body reference is taken into consideration. Instead of using the constraint equations to govern the joint behavior, the hinge degrees of freedom are explicitly embodied in the formulation. Therefore, the kinematic inconsistency (Baumgarte, 1972), encountered in numerical integration of a constrained system of differential equations can be avoided.

In section two, all the reference frames used to define the element and the joint body kinematics are introduced. Using the kinematic relations among different reference frames, the element nodal deformation with respect to the element corotational reference axes and the displacements of the element corotational axes can be written in terms of the displacements of the hinge connection points as well as hinge degrees of freedom. This is shown in section three. Using the virtual work, in section four, the generalized element internal forces which are associated with the grid point

displacements and relative hinge degrees of freedom at two ends are derived. In section five, the inertia matrix and the gyroscopic terms which include the centrifugal and Coriolis forces are then derived. Since there may be offsets from the hinge connections to the joint body reference frame, compensation procedures are taken in section six to rewrite the element equations in terms of joint body displacements. Accordingly, the equations of motion of different elements connected to the same joint body share the same generalized coordinates, namely the translational and rotational displacements of the body.

2. Reference Frames

To sufficiently depict the kinematic relations existing among joint bodies, beam elements, and hinge connections between them, in this paper, six reference coordinate systems are introduced. These six reference frames are defined as follows.

Joint Body Reference System (S_j)

S_j , which is a Cartesian coordinate system fixed to the joint body, is defined such that its axes are parallel to the global axes in the initial configuration. As it is shown in Fig.2, d_j and T_j represent, respectively, the displacement vector of the origin of the system and the 3X3 transformation matrix defined in the inertial frame. Therefore, the global displacement of an arbitrary point p^j in the joint body can be written as

$$d_p^j = d_j + T_j r_p^j - r_F^j \quad (1)$$

where r_p^j is the position of p^j in S_j .

Grid point reference system (S_g^j)

As it is shown in Fig.2, S_g^j is fixed to a grid point p_g^j on the joint body j at which an elastic member is connected through a hinge. The displacements of this system represent the motion of the grid point. This reference is defined such

that its axes are parallel to which of the associated S_j system. Therefore, one may write

$$d_g^j = d_j + T_j r_g^j - r_g^j \quad (2)$$

and

$$T_g^j = T_j \quad (3)$$

where d_g^j and T_g^j represent, respectively, the displacement vector and the orientation matrix of S_g^j , and r_g^j is the offset vector which defines the position of the grid point in the S_j system.

Grid hinge system (S_{gh}^j)

S_{gh}^j , which is also fixed to the grid point p_g^j (Fig.2), defines the orientation of the hinge by using hinge axis to be its z-axis. Therefore this reference system is rigidly attached to the grid point reference system S_g^j , and its global orientation can be represented by

$$T_{gh}^j = T_g^j {}_g\Gamma_h^j \quad (4)$$

where ${}_g\Gamma_h^j$ is the constant 3X3 matrix which describes the orientation of the grid hinge system S_{gh}^j with respect to the grid point reference system S_g^j .

Element corotational system (S_c^e)

S_c^e , whose motion represents the gross rigid motion of the element e , is defined such that its x-axis passes through two end nodes of the element, and y- and z-axes coincide with the principal axes of the cross-section of the first end node when the element is undeformed and twist with this cross-section during the dynamic motion. Standing on this coordinate system, the element's elastic behavior is observed. As it is shown in Fig.3, d_c^e and T_c^e are the translational displacement vector of the origin and the orientation matrix of this system, respectively.

Element nodal fixed system (S_e^i)

As it is shown in Fig.3, S_e^i is fixed to the i th ($i = 1$ or 2) node of the element e . The translational and rotational displacements of this system represents the motion of the i th end node. This system is defined such that the x-axis is along the tangential direction of the beam axis while the other two axes coincide with the principal axes of the cross-section of the element's end. Due to the definitions of the two systems S_c^e and S_e^i , one may relate these two systems by

$$d_e^{i=1} = d_c^e \quad (5)$$

,and due to the small nodal angular displacements resulting from the elastic deformation in the element, yields

$$T_e^i = T_c^e T_e^i \equiv T_c^e R^i \quad ; \quad i = 1, 2 \quad (6)$$

where d_e^i and T_e^i are the displacement vector and orientation matrix of system S_e^i , T_e^i is the transformation matrix representing its orientation in the element corotational system S_c^e , and

$$R^i = \begin{bmatrix} 1 & -\phi_z^i & \phi_y^i \\ \phi_z^i & 1 & -\phi_x^i \\ -\phi_y^i & \phi_x^i & 1 \end{bmatrix} \quad (7)$$

where $[\phi_x^i, \phi_y^i, \phi_z^i]^T$ is the vector of the small nodal rotations measured in the corotational system S_c^e .

Element Hinge system (S_{eh}^i)

S_{eh}^i , which is fixed at the i th end of the element e at which the element is connected to a joint body by a hinge, is defined such that the hinge axis is its z -axis. Since S_{eh}^i is also fixed at the node, there exists a constant relation between this system and its associated nodal fixed system S_{eh}^i . As it is shown in Fig.3, this relation can be written as

$$T_{eh}^i = T_e^i {}_e\Gamma_h^i \quad (8)$$

where T_{eh}^i represents the orientation of the element hinge system at the i th end node with respect to the inertial frame, and ${}_e\Gamma_h^i$ is the constant matrix describing the

kinematic relation between these two systems. The axes of the system S_{eh}^i coincide with which of the grid hinge system S_{gh}^j when the hinge displacement is zero. As it is shown in Fig.4, resulting from the hinge displacement during the dynamic process, the kinematic relation between these two system can be written as

$$T_{eh}^i = T_{gh}^j {}^jT_h^i \quad (9)$$

where ${}^jT_h^i$ represents the orientation of the element hinge system at i th end node with respect to the adjacent grid hinge system S_{gh}^j . Since S_{eh}^i rotates about its z -axis, one may write

$${}^jT_h^i = \begin{bmatrix} \cos\theta_h^{ij} & -\sin\theta_h^{ij} & 0 \\ \sin\theta_h^{ij} & \cos\theta_h^{ij} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

where θ_h^{ij} is the counter-clockwise hinge displacement from the initial configuration.

3. Element Kinematics

As it is shown in Fig.5, $i1$ and $i2$ are the two ends of element e which are connected to the joint body $j1$ and $j2$ by hinges at grid points p_g^{j1} and p_g^{j2} , respectively. Standing on the element corotational reference frame S_c^e , one may observe the small displacements at two ends due to elastic deformation by measuring the motion of the element nodal fixed systems S_e^{i1} and S_e^{i2} . Let $[\alpha_x^{i1}, \alpha_y^{i1}, \alpha_z^{i1}]^T$ and $[\phi_x^{i1}, \phi_y^{i1}, \phi_z^{i1}]^T$, and $[\alpha_x^{i2}, \alpha_y^{i2}, \alpha_z^{i2}]^T$ and $[\phi_x^{i2}, \phi_y^{i2}, \phi_z^{i2}]^T$ be the vectors of translational and rotational displacements of the two element nodal fixed reference systems with respect to the element corotational reference axes. Because of the definitions in the preceding section, one may express the constant kinematic relations between the element corotational axes and the element nodal fixed system as

$$\alpha_x^{i1} = \alpha_y^{i1} = \alpha_z^{i1} = \alpha_x^{i2} = \alpha_y^{i2} = \alpha_z^{i2} = \phi_x^{i1} = 0 \quad (11)$$

Since the length of the element changes with time, one may write

$$\alpha_x^{i2} = \| (r_o^{i2} + d_g^{j2} - r_o^{i1} - d_g^{j1}) \| - L_e \quad (12)$$

where $\|.\|$ denotes the length of a vector, r_o^{i1} and r_o^{i2} are the initial positions of the origins of the two grid point

reference frames S_g^{j1} and S_g^{j2} in the global system, and d_g^{j1} and d_g^{j2} are the translational displacements of the these two systems with respect to the inertial frame. Let

$$r_o^j = \begin{bmatrix} x_o^j & , & y_o^j & , & z_o^j \end{bmatrix}^T \quad ; \quad j = j1 , j2$$

and

$$d_g^j = \begin{bmatrix} x_g^j & , & y_g^j & , & z_g^j \end{bmatrix}^T \quad ; \quad j = j1 , j2$$

, Eq.12 can be rewritten as

$$\alpha_x^{i2} = \sqrt{(x_o^{j2} + x_g^{j2} - x_o^{j1} - x_g^{j1})^2 + (y_o^{j2} + y_g^{j2} - y_o^{j1} - y_g^{j1})^2 + (z_o^{j2} + z_g^{j2} - z_o^{j1} - z_g^{j1})^2} - L_e \quad (13)$$

According to the small rotations as defined in Eq.6, one may also write

$$\phi_y^i = t_c^{(1)T} t_e^{i(3)} \quad ; \quad i = i1 , i2 \quad (14)$$

$$\phi_z^i = - t_c^{(1)T} t_e^{i(2)} \quad ; \quad i = i1 , i2 \quad (15)$$

and

$$\phi_x^{i2} = t_c^{(3)T} t_e^{i2(2)} = (t_e^{i1(3)T} - \phi_y^{i1} t_e^{i1(1)T}) t_e^{i2(2)} \quad (16)$$

where superscripts in () refer to the number of the column matrix in the associated transformation matrix; and, according to Eqs.4, 8 and 9, one may obtain

$$\begin{aligned}
T_e^i &= \begin{bmatrix} t_e^{i(1)} & t_e^{i(2)} & t_e^{i(3)} \end{bmatrix} \\
&= T_g^j \Gamma_h^j T_h^i(\theta_h^{ij}) \Gamma_h^{iT} \quad ; \quad i = i1, i2; \quad j = j1, j2
\end{aligned}
\tag{17}$$

Therefore, in Eqs.13-16, the small nodal displacements of the element with respect to the element corotational reference axes are written in terms of the grid point translational displacement vectors d_g^{j1} and d_g^{j2} , grid point transformation matrices T_g^{j1} and T_g^{j2} , and hinge displacement θ_h^{ij} .

Since the origin of the corotational axes are fixed to the grid point p_g^{j1} , one may write the displacement vector of the corotational axes as

$$d_c^e = d_g^{j1} \tag{18}$$

And, because of Eq.6, one may write the transformation matrix of the corotational axes $T_c^e = [t_c^{e(1)}, t_c^{e(2)}, t_c^{e(3)}]$ in terms of the transformation matrix of the first element nodal fixed system S_e^{i1} as

$$t_c^{e(1)} = t_e^{i1(1)} - \phi_z^{i1} t_e^{i1(2)} + \phi_y^{i1} t_e^{i1(3)} \tag{19}$$

$$t_c^{e(2)} = t_e^{i1(2)} + \phi_z^{i1} t_e^{i1(1)} \tag{20}$$

$$t_c^{e(3)} = t_e^{i1(3)} - \phi_y^{i1} t_e^{i1(1)} \tag{21}$$

Again, as shown in Eq.17, $t_e^{i1(1)}$, $t_e^{i1(2)}$, and $t_e^{i1(3)}$ are functions of grid point transformation matrices T_g^{j1} and T_g^{j2} , and hinge displacement θ_h^{ij} . By Eqs. 18-21, the translational displacements and the orientation of the element corotational axes can be written in terms the motion of the grid point reference system as well as the hinge displacement.

4. Internal Forces

As it was defined in the preceding section, for each beam element, there is an element corotational reference frame, namely S_C^e , with respect to which a unique elastic displacement field of the element can be defined. Therefore, in this section, the element internal forces are derived using the increment of the strain energy of a deformed element.

Strain energy due to bending

In finite element formulation, the shape functions which provide the displacements along the length of the beam element in terms of displacements at the element's end nodes are established. As it was used in (Housner and *et al*, 1988), the displacement shapes for flexural motion of element e , whose two ends are $i1$ and $i2$, can be written with respect to the element corotational axes as

$$u_c^f = -y_c L_e \left[a_{i1}^{\cdot} \phi_z^{i1} + a_{i2}^{\cdot} \phi_z^{i2} \right] + z_c L_e \left[a_{i1}^{\cdot} \phi_y^{i1} + a_{i2}^{\cdot} \phi_y^{i2} \right] \quad (22)$$

$$v_c^f = L_e \left[a_{i1} \phi_z^{i1} + a_{i2} \phi_z^{i2} \right] \quad (23)$$

$$w_c^f = - L_e \left[a_{i1} \phi_y^{i1} + a_{i2} \phi_y^{i2} \right] \quad (24)$$

where u , v , w represent the displacements along x_e^C -, y_e^C -, and z_e^C -directions, respectively; x_c , y_c , and z_c are dimensionless coordinates with respect to the element length L_e along the axes parallel to the element corotational axes, $()'$ denotes first derivative with respect to x_c , and

$$a_{i1} = (x_c - 2x_c^2 + x_c^3) L_e \quad (25)$$

and

$$a_{i2} = (-x_c^2 + x_c^3) L_e \quad (26)$$

Using the Euler-Bernoulli beam, the strains due to flexural motion are

$$\epsilon_{yy}^f = \epsilon_{zz}^f = \epsilon_{xy}^f = \epsilon_{yz}^f = \epsilon_{xz}^f = 0 \quad (27)$$

and

$$\epsilon_{xx}^f = -\frac{y_c}{L_e^2} \left[a_{i1}'' \phi_z^{i1} + a_{i2}'' \phi_z^{i2} \right] + \frac{z_c}{L_e^2} \left[a_{i1}'' \phi_y^{i1} + a_{i2}'' \phi_y^{i2} \right] \quad (28)$$

By using the constitutive equation, i.e. $\sigma = E \epsilon$, one may obtain the increment of the element strain energy due to flexural motion as

$$\begin{aligned} \delta U_c^f &= \int_{V^e} \delta \epsilon_{xx}^{fT} E \epsilon_{xx}^f dv^e = L_e E \int_0^1 \int_A \delta \epsilon_{xx}^{fT} \epsilon_{xx}^f dA dx_c \\ &= \left[\delta \phi_z^{i1}, \delta \phi_z^{i2}, \delta \phi_y^{i1}, \delta \phi_y^{i2} \right] \frac{1}{L_e^3} [C_e^1] \Phi_{yz} \end{aligned} \quad (29)$$

where

$$[C_e^1] \Phi_{yz} = \begin{bmatrix} EI_z A_{11}'' & EI_z A_{12}'' & 0 & 0 \\ EI_z A_{12}'' & EI_z A_{22}'' & 0 & 0 \\ 0 & 0 & EI_y A_{11}'' & EI_y A_{12}'' \\ 0 & 0 & EI_y A_{12}'' & EI_y A_{22}'' \end{bmatrix} \begin{bmatrix} \phi_z^{11} \\ \phi_z^{12} \\ \phi_y^{11} \\ \phi_y^{12} \end{bmatrix} \quad (30)$$

in which $[C_e^1]$ is a time invariant matrix and

$$\begin{aligned} A_{11}'' &= \int_0^1 a_{i1}'' a_{i1}'' dx_c = 4 L_e^2 \\ A_{12}'' &= \int_0^1 a_{i1}'' a_{i2}'' dx_c = 2 L_e^2 \\ A_{22}'' &= \int_0^1 a_{i2}'' a_{i2}'' dx_c = 4 L_e^2 \end{aligned} \quad (31)$$

Using Eqs.14 and 15, one may obtain the generalized internal forces due to flexural motion of the element as

$$\delta U_c^f = \delta p^e \frac{1}{L_e} [B_e^1]^T [C_e^1] \Phi_{yz} \quad (32)$$

where $[B_e^1]$ is defined such that

$$\delta \Phi_{yz} = [B_e^1] \delta p^e \quad (33)$$

and

$$\delta p^e = \begin{bmatrix} \delta d_g^{j1} & \delta \pi_g^{j1} & \delta \theta_h^{11, j1} & \delta d_g^{j2} & \delta \pi_g^{j2} & \delta \theta_h^{12, j2} \end{bmatrix}$$

in which $\delta\pi_g^{j1}$ and $\delta\pi_g^{j2}$ are the vectors of virtual rotations of the two grid point reference systems.

Strain energy due to stretching

Using Eq.13, the mean value of the axial strain in the beam element can be obtained, which may be written as

$$\epsilon_{xx}^m = \alpha_x^{12} / L_e \quad (34)$$

As it was used in (Housner and *et al*, 1984), the local strain at the neutral axis of the element may be obtained by adding the first-order neutral axis strain due to flexure, which can be written as

$$\epsilon_{xx}^s = \epsilon_{xx}^m + 1/2L_e^2 \left[(v_c^f)'^2 + (w_c^f)'^2 \right] \quad (35)$$

in which $()'$ denotes the first derivative with respect to x_c . Therefore, the virtual work done by stretching of the element can be written as

$$\delta U_c^s = \int_{v^e} \delta \epsilon_{xx}^s E \epsilon_{xx}^s dv^e = EA L_e \int_0^1 \delta \epsilon_{xx}^s \epsilon_{xx}^s dx_c \quad (36)$$

By using ϵ_{xx}^m to approximate ϵ_{xx}^s , Eq.36 can be rewritten as

$$\delta U_c^s \approx \delta p^e{}^T E A \epsilon_{xx}^m L_e \left\{ [B_e^2]^T + \frac{1}{L_e^2} [C_e^2] \Phi_{yz} \right\} \quad (37)$$

where $[C_e^2]$ is a time invariant matrix

$$[C_e^2] = \begin{bmatrix} EI_z A'_{11} & EI_z A'_{12} & 0 & 0 \\ EI_z A'_{12} & EI_z A'_{22} & 0 & 0 \\ 0 & 0 & EI_y A'_{11} & EI_y A'_{12} \\ 0 & 0 & EI_y A'_{12} & EI_y A'_{22} \end{bmatrix}$$

in which

$$\begin{aligned} A'_{11} &= \int_0^1 a'_{i1} a'_{i1} dx_c = \frac{2}{15} L_e^2 \\ A'_{12} &= \int_0^1 a'_{i1} a'_{i2} dx_c = -\frac{1}{30} L_e^2 \\ A'_{22} &= \int_0^1 a'_{i2} a'_{i2} dx_c = \frac{2}{15} L_e^2 \end{aligned} \quad (38)$$

and $[B_e^2]$ is defined such that

$$\delta \alpha_x^{12} = [B_e^2] \delta p^e \quad (39)$$

Strain energy due to twisting

Because of the assumption of the linear twisting along the element axis, one may write the increment of the strain energy due to element twisting as

$$\delta U_c^t = \frac{GJ}{L_e} \delta \phi_x^{i2} \phi_x^{i2} \quad (40)$$

Using Eq.16, Eq.40 can be rewritten as

$$\delta U_c^t = \delta p^e{}^T \left\{ \frac{GJ}{L_e} [B_e^3]^T \phi_x^{i2} \right\} \quad (41)$$

in which $[B_e^3]$ is defined such that

$$\delta \phi_x^{i2} = [B_e^3] \delta p^e \quad (42)$$

According to Eqs.32, 37 and 41, the virtual work done by the total generalized internal forces which include the effects of element bending, stretching, and twisting can be written as

$$\delta w_i^e = \delta U_i^e = \delta U_c^f + \delta U_c^s + \delta U_c^t = \delta p^e{}^T F_i^e \quad (43)$$

The total element internal forces F_i^e are thus obtained.

5. Inertia

In the preceding section the generalized internal forces due to the deformation of an element e are derived. In this section the derivation of the inertia of the deformable element resulting from the element large motion as well as elastic displacements is presented. Combining the flexural, stretching, and twisting effects, the total elastic displacement field of an element with respect to the element corotational axes can be written as

$$u_c^e = N_c^e \psi_c^e \quad (44)$$

where

$$N_c^e = \begin{bmatrix} z_c L_e a_{i1}' & -y_c L_e a_{i1}' & z_c L_e a_{i2}' & -y_c L_e a_{i1}' & 0 & x_c \\ 0 & L_e a_{i1} & 0 & L_e a_{i2} & -z_c L_e & 0 \\ -L_e a_{i1} & 0 & -L_e a_{i2} & 0 & y_c L_e & 0 \end{bmatrix} \quad (45)$$

and

$$\psi_c^e{}^T = [\phi_y^{i1}, \phi_z^{i1}, \phi_y^{i2}, \phi_z^{i2}, \phi_x^{i2}, \alpha_x^{i2}] \quad (46)$$

Therefore, the total displacement of an arbitrary point in element e can be written as

$$u_g^e = d_c^e + L_e (T_c^e - T_c^o) r_c + T_c^e N_c^e \psi_c^e \quad (47)$$

where $()^0$ denotes the initial configuration, and $r_c^T = [x_c, y_c, z_c]$. Using Eq.18, and Eqs.4, 6, 8, and 9; respectively, d_c^e and T_c^e can be written as

$$d_c^e = d_g^{j1} \quad (48)$$

and

$$T_c^e = - T_g^{j1} {}_g\Gamma_h^{j1} {}_{j1}T_h^{i1} {}_e\Gamma_h^{i1^T} R^{i1} \quad (49)$$

Let

$$r_c^* = L_e r_c + N_c^e \psi_c^e$$

$$S_c = - T_g^{j1} {}_g\Gamma_h^{j1} {}_{j1}T_h^{i1} {}_e\Gamma_h^{i1^T}$$

$$S_c' = - T_g^{j1} {}_g\Gamma_h^{j1} \frac{\partial {}_{j1}T_h^{i1}}{\partial \theta_h^{i1, j1}} {}_e\Gamma_h^{i1^T}$$

$$S_c'' = - T_g^{j1} {}_g\Gamma_h^{j1} \frac{\partial^2 {}_{j1}T_h^{i1}}{\partial \theta_h^{i1, j1^2}} {}_e\Gamma_h^{i1^T}$$

and

$$T_c' = S_c' R^{i1}$$

$$T_c'' = S_c'' R^{i1}$$

, the virtual displacements of any arbitrary point on element e can be written as

$$\begin{aligned}
\delta u_g^e &= [B_c^e] \delta p^e \\
&= \left\{ \frac{\partial d_g^{j1}}{\partial p} - \left(\overline{T_c^e r_c^*} \right) \frac{\partial \pi_g^{j1}}{\partial p^e} + \left(T_c^e r_c^* \right) \frac{\partial \theta_h^{i1, j1}}{\partial p^e} \right. \\
&\quad \left. - S_c \left(\overline{R^{i1} r_c^*} \right) \frac{\partial \phi_c^{i1}}{\partial p^e} + T_c^e N_c^e \frac{\partial \psi_c^e}{\partial p^e} \right\} \delta p^e
\end{aligned} \tag{50}$$

in which

$$\phi_c^{i1} = [0, \phi_y^{i1}, \phi_z^{i1}]^T$$

Similarly,

$$\dot{u}_g^e = [B_c^e] \dot{p}^e \tag{51}$$

Taking time derivative again, yields

$$\ddot{u}_g^e = [B_c^e] \ddot{p}^e + b_c^e \tag{52}$$

where

$$\begin{aligned}
b_c^e &= \tilde{\omega}_g^{j1} \{ \tilde{\omega}_g^{j1} T_c^e r_c^* + \theta_h^{i1, j1} T_c^e r_c^* + S_c \tilde{\phi}_c^{i1} R^{i1} r_c^* \} \\
&\quad + \theta_h^{i1, j1} \{ \tilde{\omega}_g^{j1} T_c^e r_c^* + \theta_h^{i1, j1} T_c^e r_c^* + S_c \tilde{\phi}_c^{i1} R^{i1} r_c^* \} \\
&\quad + \{ S_c^{i1, j1} \tilde{\phi}_c^{i1} + \omega_g^{j1} S_c + \theta_h^{i1, j1} \} \tilde{\phi}_c^{i1} R^{i1} r_c^*
\end{aligned}$$

$$+ 2 \{ \omega_g^{j1} T_c^e N_c^e \dot{\psi}_c^e + \dot{\theta}_h^{i1,j1} T_c^e N_c^e \dot{\psi}_c^e + S_c \dot{\phi}_c^{i1} R^{i1} N_c^e \dot{\psi}_c^e \} \quad (53)$$

Since the virtual work done by the inertial forces of element e may be expressed as

$$\delta W_m^e = \rho^e L_e \int_0^1 \delta u_q^{eT} \ddot{u}_q^e dx_c \quad (54)$$

where ρ^e is the mass density; using Eqs.50 and 52, one may write

$$\delta W_m^e = \delta p^{eT} [M^e] \ddot{p}^e + \delta p^{eT} g^e \quad (55)$$

where $[M^e]$ is the element inertia matrix and g^e is a vector of quadratic velocity terms which include the centrifugal and Coriolis forces of the element, which can be written, respectively, as

$$[M^e] = \rho^e L_e \int_0^1 [B_c^e]^T [B_c^e] dx_c \quad (56)$$

and

$$g^e = \rho^e L_e \int_0^1 [B_c^e]^T b_c^e dx_c \quad (57)$$

Using Eq.43 and 55, the total virtual work of the element can be written in matrix form as

$$\delta W_T^e = \delta p^{e^T} \{ [M^e] \ddot{p}^e + g^e + F_i^e - Q^e \} = 0 \quad (58)$$

where Q^e is the vector of generalized external forces which can be obtained by using the virtual work.

6. Offset Compensation and Connectivity

In the previous sections, the element equations of motion are derived in the generalized space spanned by the displacements (translational and rotational) of the grid point reference systems S_g^{j1} and S_g^{j2} on the joint bodies where the element is connected, as well as the hinge degrees of freedom existing between joint bodies and the element. In many cases, a joint body may be used to connect several elements. Using the displacements of each connecting grids on the same joint body to be the system's generalized coordinates, the efficiency in solving the dynamic equations must be decreased due to the increased number of dependent generalized coordinates. Therefore, in this section, a so-called offset compensation method is introduced. On each joint body, only one grid point, which will be referred to as the primary grid point, is used to represent the motion of the joint body. All the other grid points on the joint body where elements are connected are referred to as the offset points to the primary grid point. Using the PVW, the equations of motion of each element connected to the offset points can be rewritten in terms of the motion of the primary grid point as well as the hinge displacements. The system's equations of motion can be thus assembled by using the minimum number of generalized coordinates.

Using the kinematic relations between an offset point and its associated primary grid, which is as written in Eqs.2 and 3, it yields

$$\delta p^e = [L^e] \delta q^e \quad (59)$$

where

$$\delta q^{e^T} = \begin{bmatrix} \delta d_{j1} & \delta \pi_{j1} & \delta \theta_h^{i1, j1} & \delta d_{j2} & \delta \pi_{j2} & \delta \theta_h^{i2, j2} \end{bmatrix}$$

and

$$[L^e] = \begin{bmatrix} I_3 & \overbrace{-(T_{j1} r_g^{j1})} & 0 & & & \\ 0 & I_3 & 0 & & 0_6 & \\ 0 & 0 & 1 & & & \\ & & & I_3 & \overbrace{-(T_{j2} r_g^{j2})} & 0 \\ & 0_6 & & 0 & I_3 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

in which 0_6 's are 6X6 null matrices, and I_3 are 3X3 identity matrices. Similarly,

$$\dot{p}^e = [L^e] \dot{q}^e \quad (60)$$

Taking derivative with respect to time, one may get

$$\ddot{p}^e = [L^e] \ddot{q}^e + \chi^e(q, \dot{q}) \quad (61)$$

where

$$\chi^e = \begin{bmatrix} -\tilde{\omega}^{j1} \tilde{\omega}^{j1} \overline{(T_{j1} r_g^{j1})} \\ 0 \\ 0 \\ -\tilde{\omega}^{j2} \tilde{\omega}^{j2} \overline{(T_{j2} r_g^{j2})} \\ 0 \\ 0 \end{bmatrix}$$

, in which ω 's are vectors of angular velocities.

Substituting Eqs.59 and 61 into Eq.58, the element equations can be written as

$$[\bar{M}]^e \ddot{q}^e = \bar{F}^e \quad (62)$$

in which

$$[\bar{M}]^e = [L^e]^T [M^e] [L^e]$$

and

$$[\bar{F}]^e = [L^e]^T (Q^e - F_1^e - g^e - [M^e] \chi^e)$$

Since all the elements connected to a joint body share the same generalized coordinates, which are the translational and rotational displacements of the joint body, the system equations of motion can be assembled by the element equations of Eq.62 using the minimum number of generalized coordinates with minimum number of constraint equations.

7. Summary

In this paper, a finite element approach is used to formulate the dynamic equations of joint-dominated flexible multibody systems undergoing large rotations. On each element, a set of element corotational axes are selected to represent the rigid modes of the element motion. Using the element kinematics defined in the paper, the element rigid modes and the small elastic deformation with respect to the element corotational reference can be written in terms of the displacements of the grid points on the joint bodies, at which the two ends of the element are connected through hinges, and the associated hinge rotations. Using the PVW (principle of virtual work) the element equations of motion can be derived. In this method, the hinge connection is defined by introducing a relative degree of freedom instead of using the constraint equations which may cause the system configuration inconsistency due to the truncation errors accompanied with the numerical integrations. To further reduce the number of generalized coordinates in the system, the offset compensation method is used for the element connectivity at the joint body. Based on the formulations presented in the paper, a general purpose transient dynamics code LATDYN (Large Angle Transient Dynamics) has been developed in NASA Langley and used for the design and analysis of space crafts and structures. Even though only beam elements and hinge joints are used presently, the method

presented in this paper may also be applied to more general systems consisting of different elements as well as various joint connections. This is worth further studies.

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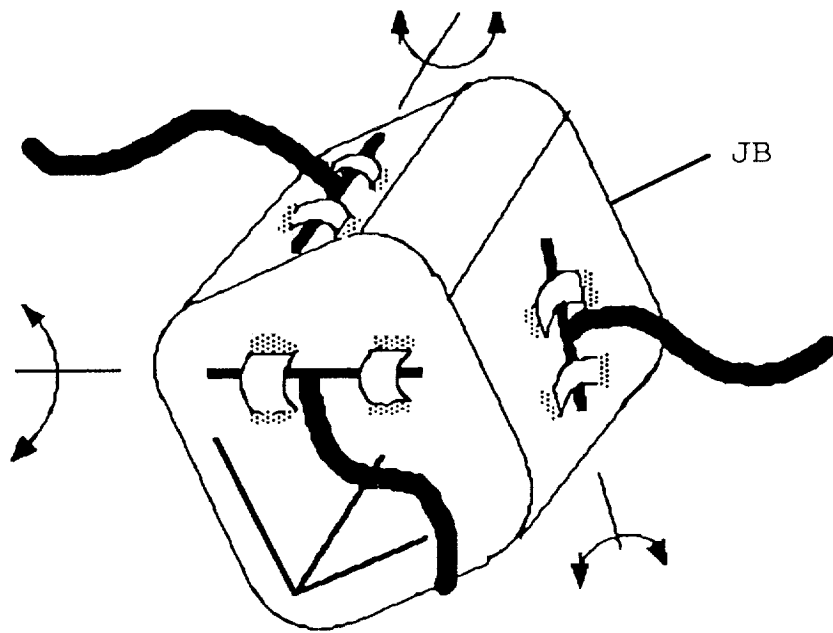


Figure 1 : Multi-Hinge Joint Body

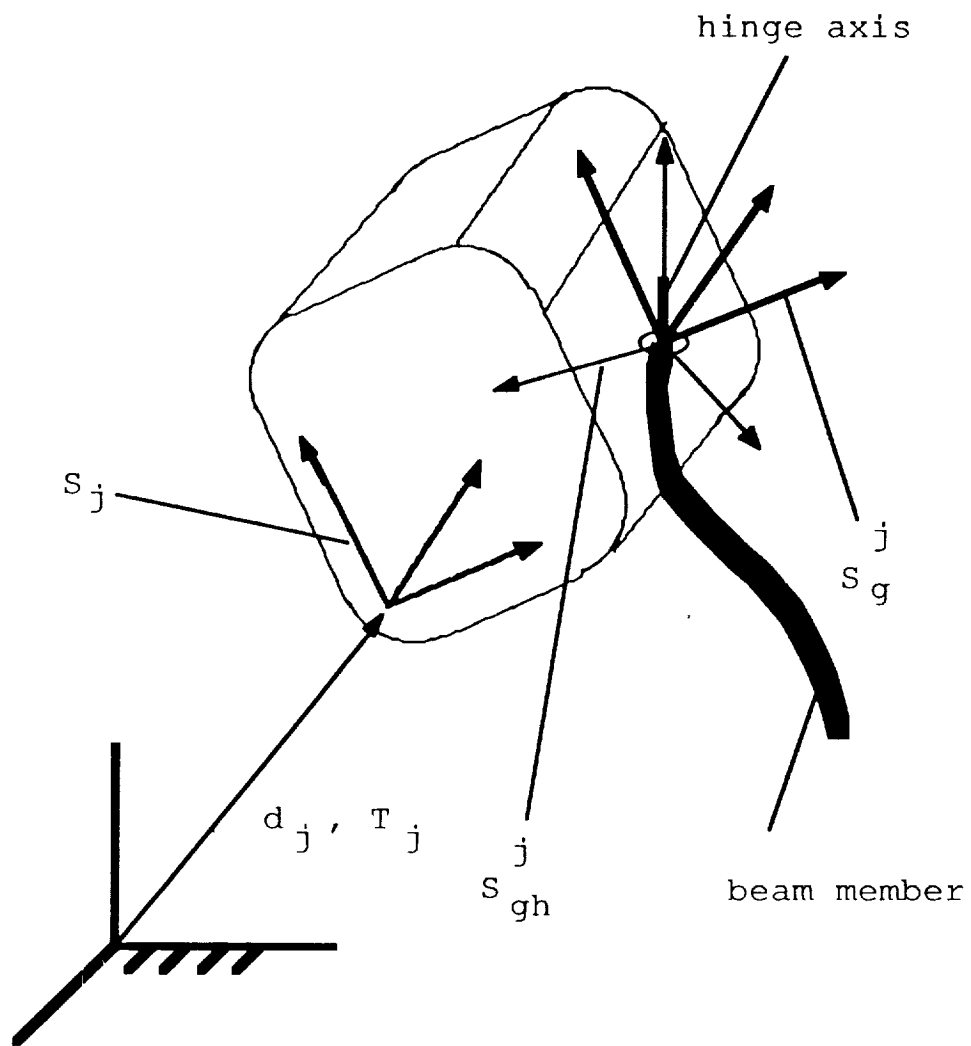


Figure 2 : Reference Frames on Joint Body

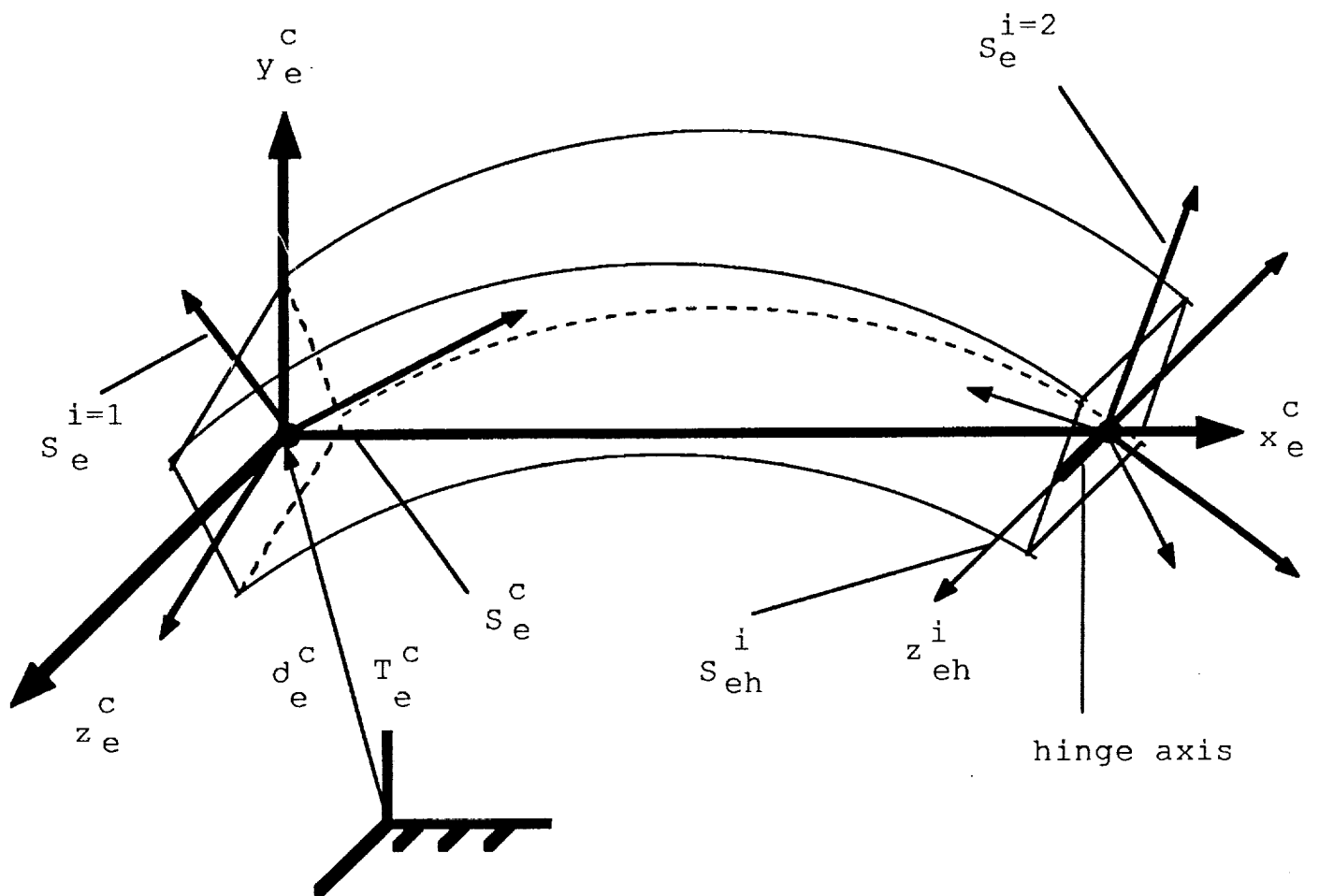


Figure 3 : Reference frames on a Beam Element

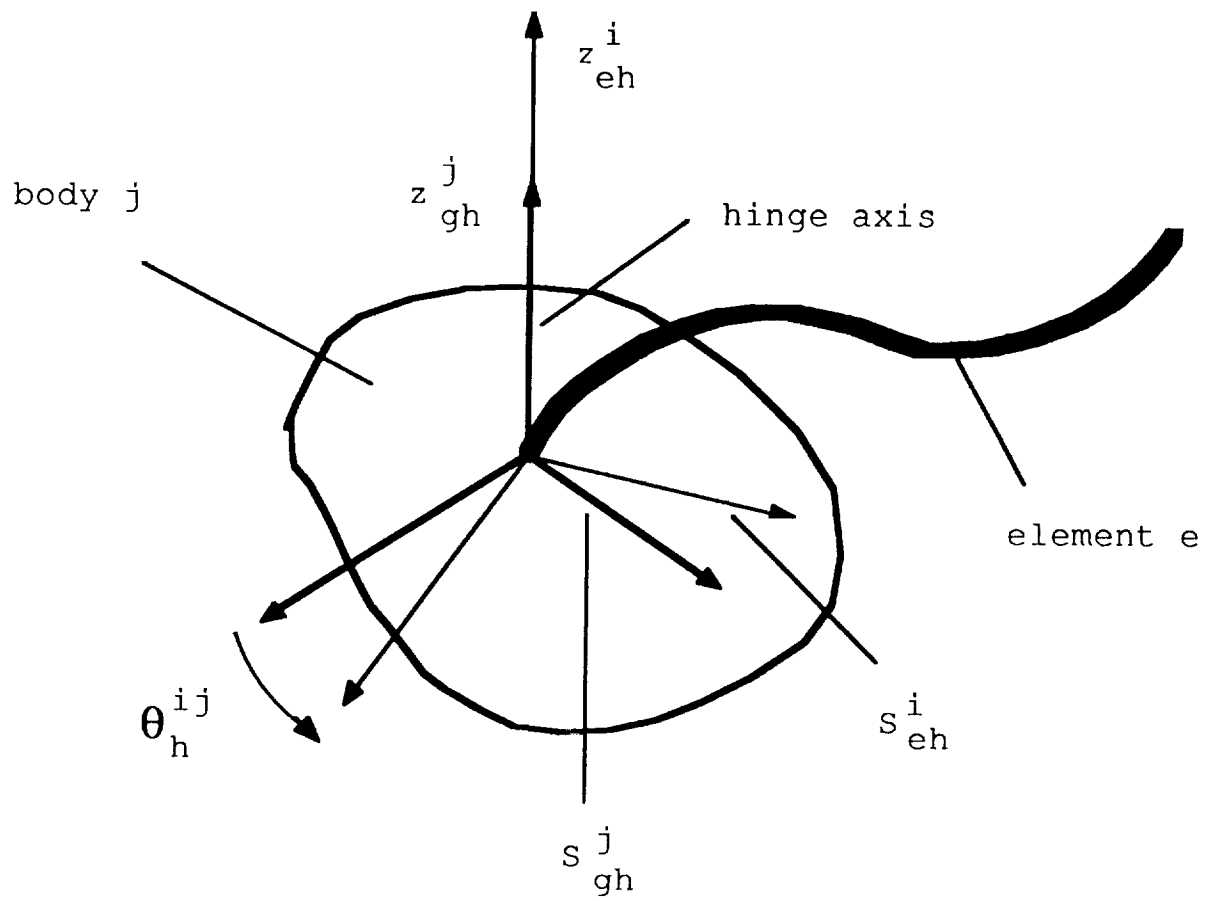


Figure 4 : Hinge Degree-of-Freedom

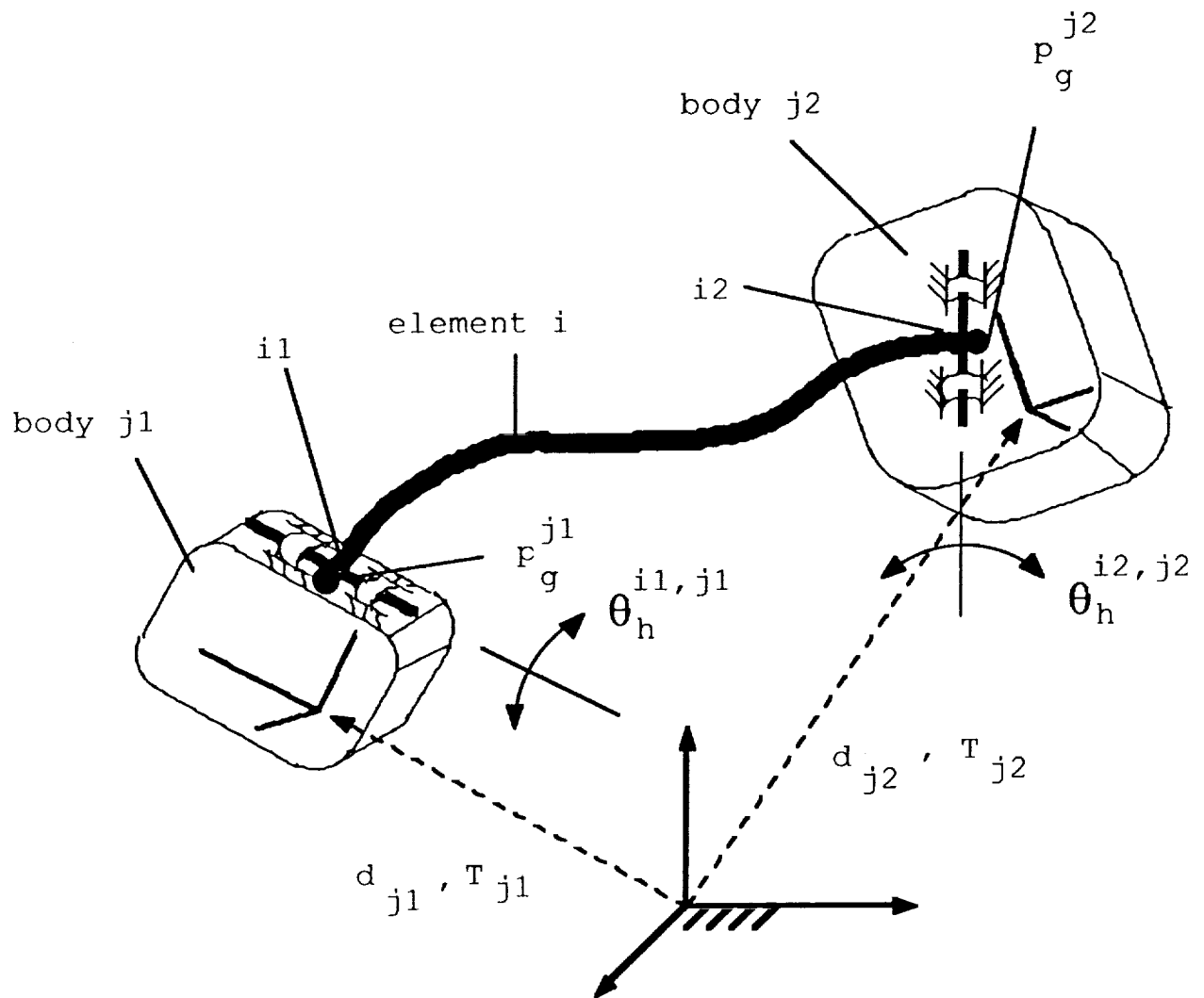


Figure 5 : General Hinge-Connected Beam Element



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16. Abstract A finite element method to model dynamic structural systems undergoing large rotations is presented. The dynamic systems are composed of rigid joint bodies and flexible beam elements. The configurations of these systems are subject to change due to the relative motion in the joints among interconnected elastic beams. A body fixed reference is defined for each joint body to describe the joint body's displacements. Using the finite element method and the kinematic relations between each flexible element and its corotational reference, the total displacement field of an element, which contains gross rigid as well as elastic effects, can be derived in terms of the translational and rotational displacements of the two end nodes. If one end of an element is hinged to a joint body, the joint body's displacements and the hinge degree of freedom at the end are used to represent the nodal displacements. This results in a highly coupled system of differential equations written in terms of hinge degrees of freedom as well as the rotational and translational displacements of joint bodies and element nodes.			
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